Problem 3: Power Function - Solution

Note that as the empty set is contained in every set, we have $\emptyset \subseteq f(\emptyset)$. Then, as f is order preserving, we get from repeatedly applying f a sequence:

$$\emptyset \subseteq f(\emptyset) \subseteq f(f(\emptyset)) \subseteq ...$$

We call its elements A_i with $i \in \mathbb{N}$. Remark that either $A_{i+1} = f(A_i) = A_i$ or it is larger. If it is equal, we are done. Otherwise, the size of the sets of our sequence increases. But, since $[n]$ has size n, we need at most $n+1$ queries before we find a stationary point, which is less than $n+3$.