Problem : Grazing Goats - Solution

Note that each goat grazes a perfect circle (disk) with radius r_i, so that we need to calculate the overlap of n circles. For $n = 1$, the answer simply is πr_i^2, but when circles overlap, the problem becomes much more difficult. For $n = 2$ overlapping circles, the key is to note that we can split the figure into two circle arcs and a quadrilateral, as in Figure 1. We can then calculate the area of the figure by calculating the area of the arcs and the quadrilateral separately.

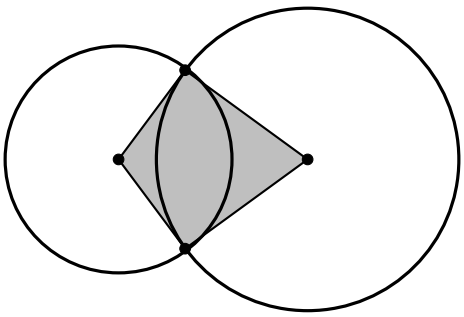

Figure 1: Two overlapping circles

Note that the vertices of the polygon in the decomposition alternate between intersection points and midpoints of the circles. With more circles, the idea remains the same; we again want to decompose the figure into disjoint polygons and circle arcs, of which we can calculate the area much more easily. However, finding the arcs and polygons (each circle can now contribute to more than one arc and polygon) again becomes more difficult. An example of such a decomposition for $n = 4$ circles can be seen in Figure 2.

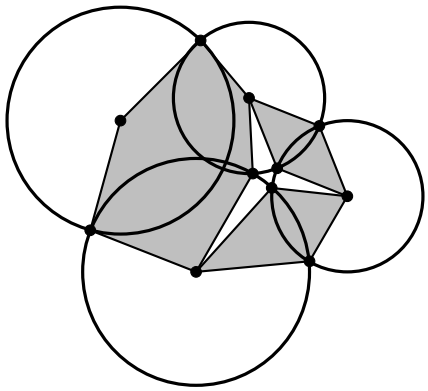

Figure 2: More than two overlapping circles

In most cases, finding such a decomposition by hand is quite feasible for a specific case, but we need to derive an algorithm that works for any case. For this, we first determine the vertices of the polygons in our decomposition. These vertices are the intersection points of two circles, that do not lie on the interior of any of the other circles. This can be done efficiently through a sweep-line algorithm, but as $n \leq 100$, the basic quadratic approach of checking every pair of circles suffices. What remains is to construct the decomposition into polygons and arches. What works is to alternate each polygon between midpoints of the circles and intersection points; any intersection point has the two corresponding midpoints as neighbouring vertices, and for each midpoint the neighbouring intersection points in the polygons of the decomposition are adjacent on the circle. Thus, we can find the correct decomposition by picking a midpoint for which not all intersection points have been visited, picking the next intersection point along the circle in a clockwise direction, and continuing until a cycle has been found; this is one of the polygons. Iterate until all polygons are found; the arcs that need to be added are the arcs between pairs of intersection points that are not part of the same polygon. This can be done during construction of the different polygons. See the solution code for implementation details.