μ - Games

Mathematics Utrecht

Utrecht University

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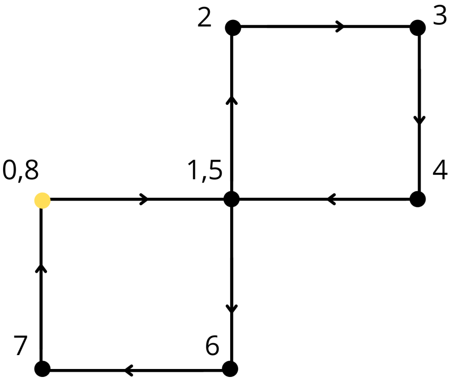
Closer look at example:



We take $n/2$ steps up/down and $n/2$ to left/right.

To return to origin: n/4 steps up/down/left/right.

So: if $n \neq 0 \mod 4$, we have 0 paths.

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Explanation: $S = \{1, 2, 3\}$, then all possible subsets are:
$2^S = \{\{1, 2, 3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1\}, \{2\} \{3\}, \{\}\}$
One can see that $S_1 = \{1\}, S_2 = \{2\}, S_3 = \{3\}$ is the largest family of subsets, hence m=3.

For any set of n elements, we can take $S_1, ..., S_n$ as singleton sets. This is a valid distribution. So $m \geq n$.

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Proof: m$\leq n$

Let $S_1, ..., S_m$ some distribution of subsets of S. Define the matrix over $\mathbb{F}_2$:

$$A_{ij} = \begin{cases} 1 & \text{if } j \in C_i \\ 0 & \text{else} \end{cases}.$$

Note that the rank of A is at most n as we are working over $\mathbb{F}_2$. Then take $B = A_{ij}A^T$. By the rules, we have that $B = I_m$. We know that for the rank it holds:

$$rank(B) \leq min(rank(A), rank(A^T)) = n$$

So $m \leq n$.

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Unstable Matrix

We define $p_n = det(F_n - \lambda\mathbb{I}_n)$.

Take the Laplace expansion of $F_n - \lambda\mathbb{I}$. This gives

$$p_n = (1-\lambda)p_{n-1} - (n-1)\lambda p_{n-2}.$$

We can use this to show

$$p_n(\lambda^{-1}) = (-1)^n\lambda^{-n}p_n(\lambda).$$

Hence, the coefficient spectrum is palindromic when n is even and anti-palindromic when n is odd.

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By taking the substitution $x \mapsto \sqrt{t}x$, we get

$$\int x^k \sigma_t(x) dx = t^{k/2} \int x^k \sigma_1(x)\ dx.$$

Note that for odd k, this integral equals zero.
Now look at $2k$ for $k \in \mathbb{N}_{>0}$. We can calculate $\int x^k \sigma_1(x)\ dx$ by making the substitution $x = 2\sin\theta$. This gives

$$\begin{aligned}\int x^{2k}\sigma_1(x)\ dx &= 2^{2k+1}\left[\prod_{l=1}^{k}\frac{2l-1}{2l} - \prod_{l=1}^{k+1}\frac{2l-1}{2l}\right] \\ &= C_k.\end{aligned}$$

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Important observation!
A point $x \in \mathbb{R}^n$ is a limit point of these equations if and only if $\{v_i \mid x_i = 1\}$ is a maximal independent set in the graph G.

We are looking at configurations of the combinations 10 and 100, such that in total we end up with n numbers. So we are looking for x and y such that

$$2x + 3y = n.$$

We denote the function that returns this number of configurations by $f(n)$.

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Note we have the following cases.

- We start with 1, end with 0. The number of cases is given by $f(n)$.
- We start with 01, end with 1. The number of cases is given by $f(n-2)$.
- We start with 01, end with 10. The number of cases is given by $f(n-3)$.
- We start with 001, end with 1. The number of cases is given by $f(n-3)$.

So the total answer is given by Total $= f(n) + f(n-2) + f(n-3) + f(n-3)$.
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