# Problem 2: Odd Sets Solution

**Proof:** $m \geq n$

Explanation: $S = \{1,2,3\}$, then all possible subsets are: $2^S = \{\{1,2,3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1\}, \{2\}\ \{3\}, \{\}\}$
One can see that $S_1 = \{1\}, S_2 = \{2\}, S_3 = \{3\}$ is the largest family of subsets, hence m=3.
For any set of $n$ elements, we can take $S_1, ..., S_n$ as singleton sets. This is a valid distribution. So $m \geq n$.

**Proof:** $m \leq n$
Let $S_1, ..., S_m$ some distribution of subsets of $S$. Define the matrix over $\mathbb{F}_2$:

$$A_{ij} = \begin{cases} 1 & \text{if } j \in C_i \\ 0 & \text{else} \end{cases}.$$

Note that the rank of $A$ is at most $n$ as we are working over $\mathbb{F}_2$. Then take $B = A_{ij}A^T$. By the rules, we have that $B = I_m$. We know that for the rank it holds:

$$rank(B) \leq min(rank(A), rank(A^T)) = n$$

So $m \leq n$.