Problem 3: - Solution

Let us take a look at what each pirate takes. The first pirate takes his fraction: $m \cdot \frac{p}{q}$. Then what is left is $m - m \cdot \frac{p}{q}$. Therefore the second pirate takes
$(m - m \cdot \frac{p}{q}) \cdot \frac{p}{q}$. Then what is left is $m - m \cdot \frac{p}{q} - (m - m \cdot \frac{p}{q}) \cdot \frac{p}{q} = m(1 - \frac{p}{q})^2$.
As we continue this way, we see that the ith time a pirate gets money, he/she will get $(m - (1 - \frac{p}{q})^{i-1})\frac{p}{q}$.
Therefore we see that the first pirate in total will get:

$$\sum_{i=0}^{\infty}(m(1 - \frac{p}{q})^{n \cdot i})\frac{p}{q}$$

We see that the $j - th$ pirate (provided the first pirate is pirate 0) will get:

$$\sum_{i=0}^{\infty}(m(1 - \frac{p}{q})^{n \cdot i + j})\frac{p}{q}$$

In this, we recognize a geometric series. So we see that this is equal to

$$\frac{\frac{mp}{q} \cdot (1 - \frac{p}{q})^j}{1 - (1 - \frac{p}{q})^n}.$$

Now for this to be a natural number, we see that $1 - (1 - \frac{p}{q})^n$ should divide $\frac{mp}{q} \cdot (1 - \frac{p}{q})^j$ for all $j \in \{0, 1, ..., n - 1\}$. We see that when this holds for $j = n - 1$, this will hold for all j.
We can rewrite $1 - (1 - \frac{p}{q})^n$ divided by $\frac{mp}{q} \cdot (1 - \frac{p}{q})^j$ to:

$$\frac{mp(q - p)^{n-(j+1)}}{q^n - (q - p)^n}.$$

This should be a natural number for every j. We see that when this holds for $j = n - 1$, this will hold for all j.
Therefore we only have to check whether:

$$\frac{mp}{q^n - (q - p)^n}.$$

Now to find the smallest q and the smallest p, we can just loop over them and check whether the fraction is a natural number.