# Problem 2: Weierstrass Fixed Point - Solution

Firstly, we can reformulate the problem of find a fixed point of $f$, as finding a zero of the function $g(x) = f(x) - x$. Now, we observe that as the derivative of the Weierstrass function does not exist, we cannot use methods such as Newton-Raphson to find the zero of $g$.

In stead, we will use binary search. Let $a = 0$ and $b = \pi$. We observe that $g(0) = f(0) = \sum a^n > 0$ and $g(\pi) = f(\pi) - \pi = -\sum a^n - \pi < 0$. Now, let $m = (a+b)/2$. Then, either $g(m) = 0$ i.e. $m$ is a fixed point, or $g(m) > 0$, so we can set $a = m$ or $g(m) < 0$, so we can set $b = m$ after which we still satisfy the conditions that $f(a) > 0 > f(b)$. Iterating this, we get a sequence of $a_n$ and $b_n$ such that $f(a_n) > 0 > f(b_n)$. Now, by the intermediate value theorem, there exists and $x_n$ in between $a_n$ and $b_n$ such that $f(x_n) = 0$. Furthermore, we note that $[a, b]$ is a closed interval and $f$ is continuous. So, $f$ is uniformly continuous. So, for every $\epsilon > 0$ there exists a $\delta > 0$ such that $|x - y| < \delta$ implies that $|g(x) - g(y)| < \epsilon$. Now, as $|a_n - b_n| < \delta$ eventually and $x_n \in [a_n, b_n]$. It follows that eventually our points in our interval are sufficiently close to the fixed point that they themselves are approximately fixed points.

Finally, one thing to note is that on needs to calculate the Weierstrass function with sufficient accuracy. However, we note that this is not problematic as we easily bound the error of our calculation by noting that the residual sum $\sum_{n=k}^{\infty} a^n = a^n/(1-a)$.