MATHEMATICS DEPARTMENT

"A computer is the mathematicians best friend"

μ - Games
Mathematics Utrecht

April 2024

Rules:

The idea of this event is to gap the bridge between mathematics and programming. When working on these exercises, we hope the participant will get a better understanding of the underlying mathematical concepts. You will not be required to do a lot of difficult programming. With array manipulation and basic functionality, you should be able to solve all the exercises.

When working on these exercises, you must conform to the following rules.

- You are allowed to work in groups of maximum 4 persons.
- You will have 3 hours to solve the problems.
- For the problems, you can use the default mathematics library of your programming language (for example *import fractions* in Python).
- You cannot look up any computer code that may help you with solving the problem.

After 3 hours, the solutions to the exercises will be discussed. To check your own solution, one can go to the website `http://clover.science.uu.nl/dj`.

Problem 1: Cross-Country Circus

Difficulty: ★ ☆ ☆ ☆ ☆
Key words: Geometry, Topography

Carla is one of the most legendary tightrope walkers alive today and she is planning on performing another record-breaking stunt next year. Her plan is to walk on a tightrope across *two entire countries*, Abyssnia and Balantia. Carla has asked you to deal with her visa applications for both countries, since she will be busy practicing, and filling out visa application forms while high above the Grand Canyon is an entirely different feat, one which she is only planning to do in the year after her cross-country circus performance.

Conveniently, the two countries of Abyssnia and Balantia are perfectly circular, and touch in a single point, in a place called Crossing-Bordernia. To avoid dealing with any other bordering countries, Carla ties her tightrope in such a way that it crosses the border precisely above Crossing-Bordernia. Due to the good relations between Abyssnia and Balantia, Carla only needs a single visum, with a specification for what fraction of the time she will be in Abyssnia. Thus, you only need to calculate this ratio. As Carla is an amazing acrobat, she walks along the tightrope with a constant speed.

In Figure 1, the first sample case is illustrated. Here, Carla spends precisely half her time in Abyssnia.

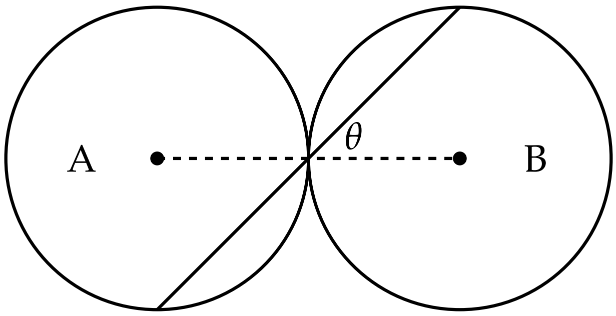


Figure 1: Sample case 1.

Input

- The first line contains two space-separated integers $0 < r_A, r_B \leq 10^5$, the radii in meters of the countries of Abyssnia and Balantia respectively.
- The second line contains a single integer $-90 < \theta < 90$, the angle in degrees Carla's tightrope makes with the straight road between the capitals of Abyssnia and Balantia, located at their respective midpoints.

Output

- The fraction of time Carla spends in Abyssnia. Your answer should have a relative error of at most 10^{-6}.

Examples

Input	Output
1000 1000 45	0.5

Input	Output
655 1834 13	0.263157894

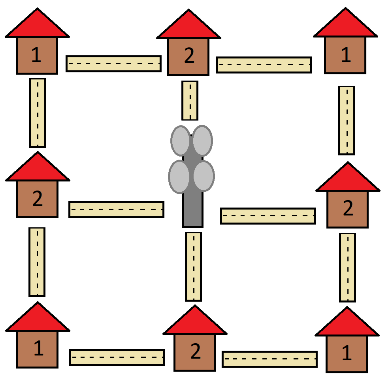
Problem 2: Strong Signal

Difficulty: ★ ☆ ☆ ☆ ☆
Key words: Manhattan distance, Tiling

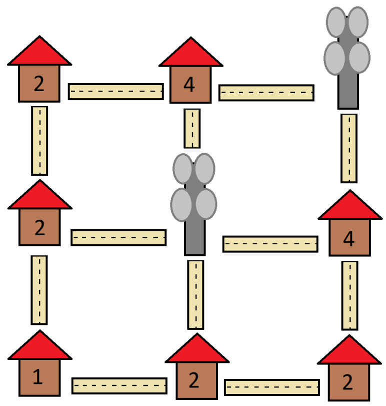
The Square City life is amazing! It's layout is very squary, so you can easily get anywhere. One minor point though: they don't have any cell towers. Therefore, the city council has decided to replace some houses by cell towers. The signal at all the remaining houses should have strength two at least. The signal strength given by a tower i to a house j equals

$$\max\{3 - d(i,j), 0\},$$

where $d(i,j)$ denotes the distance between the tower and the house. In this case, the distance equals the Manhattan distance, which is the amount of roads one has to walk to go from the tower to the house.



(a) An example of the signal strength of houses with one tower in the city.



(b) The signal strength in the houses where the signals of towers are added.

Of course, the city council wants to destroy as few homes as possible and keep costs low. What is the smallest number of towers that can ensure a solid signal at all of the houses in a town of $3 \times n$?

Input

- An integer $1 \leq n \leq 10^8$, representing the width of the grid.

Output

- The minimum amount of telephone towers needed to give every spot on the map a signal of at least strength 2.

Examples

Input	Output
3	2

Input	Output
4	3

Problem 3: Murky Maze

Difficulty: ★ ★ ☆ ☆ ☆
Key words: Interactive, Maze

Midas, a daring thief, attempted to steal the wand of an evil wizard but was caught. The evil wizard took Midas's eyesight and threw him into a maze. Now, he needs your help to escape. At each spot, Midas can only feel the directions he can move: N (North), E (East), S (South), and W (West). Each turn he tells you the directions he can move in, upon which you should respond with a direction. To prevent starvation, Midas should exit the maze within $2m$ turns, where m is the number of nodes in the maze. It is guaranteed that all the walls are connected.

Interaction

This is an interactive problem. Your submission will be run against an *interactor*, which reads from the standard output of your submission and writes to the standard input of your submission. This interaction needs to follow a specific protocol:

Each turn, the interactor sends a single line, containing either the string "`Finally, freedom!`" or 1 to 4 letters from the set {N, E, S, W} indicating which directions one can move in. In the first case, Midas has reached the exit, upon which your program should terminate. In the second case, your program should print one of the available directions to indicate what direction Midas should move in.

Sending any output after the interactor has replied with "`Finally, freedom!`" or taking more than $2m$ turns results in a wrong answer.

Make sure you flush the buffer after each write.

Examples

In the example, we have put each in/output on its own line, so that it is clear in what order the in and output is given.

Read	Write
N S	
	S
N W	
	N
N S	
	N
Finally, freedom!	

Problem 4: Binary Beauty

Difficulty: ★ ★ ★ ☆ ☆
Key words: Combinatorics

Due to some unfortunate events with a curling iron, Rapunzel has once again lost most of her hair, having "only" 1 metre left. Of course, she would like her locks back as soon as possible and thus she has turned to some dubious hair products. One of the products will add 1 metre to her hair overnight, while the other will double her hair length overnight. She can only use one product each day, and would like to get her hair back *precisely* to its original length as soon as possible. Can you help her out?

Input

- A single integer $2 \leq n < 10^{100}$, Rapunzel's target hair length in metres.

Output

- On the first line a single integer m, the minimum number of days for Rapunzel to get her hair back.
- Next, m lines of which the ith is either `ADD` or `DOUBLE`, depending on the hair product which Rapunzel should use on the ith day to get to her target length of n. If there are multiple valid answers, you may print any one of them.

Examples

Input	Output
2	1 ADD

Input	Output
5	3 ADD DOUBLE ADD

Problem 5: Quantum Quiz

Difficulty: ★ ★ ★ ★ ☆
Key words: Quantum Logic, Kripke Models

Quentin has recently become convinced that Quantum Logic contains a solution to all the problems of the world. So, in order to solve world hunger, he wants to better understand Quantum Logic. In particular, orthologic.

To model such logics, he considers a so called *orthoframe*. This consists of a set of worlds I and a reflexive symmetric accessibility relation $\not\perp$ on I, whose negation we denote by $\perp$. Now, a set of worlds $X \subseteq I$ has a complement

$$X' = \{i \in I \mid \forall j \in X : i \perp j\}.$$

That is X' is the set of worlds inaccessible to X. In the case $j \in X'$, we write $j \perp X$. Now, a subset $X \subseteq I$ is called a proposition if

$$i \in X \iff \forall j \in I : (i \not\perp j \implies j \not\perp X).$$

This is the starting point of providing a semantic model of our logic.[1]

Quentin wants to know given a world of n objects and a reflexive, symmetric relation $\not\perp$ and a set X, what is the smallest proposition containing X. (Note that I itself is a proposition, so we always have a smallest proposition containing X for I finite.) The relation $\not\perp$ is specified through a minimal collection of pairs (i, j) with $i, j \in I$ generating $\not\perp$. In this case, by generating, we mean that $\not\perp$ is the smallest reflexive symmetric relation such that $i \not\perp j$ for all the specified pairs i and j. In particular, this means that $i \not\perp i$ is implicitly assumed to be related and if i is indicated to be related to j, we also have that j is related to i.

Input

- $1 \leq n \leq 10^3$ the number of worlds.
- $1 \leq m \leq 10^6$ the number of pairs to generate the relation.
- m lines with on each line a pair i j ($1 \leq i, j \leq n$) indicating i and j are related.
- $1 \leq l \leq 10^6$ representing the size of the set X.
- l distinct sorted integers $x_1, ..., x_l$ specifying the set X.

Output

- A number k representing the size of the output set.
- k distinct sorted integers $y_1, ..., y_k$ specifying the smallest proposition Y such that $X \subseteq Y$.

Examples

Input	Output
5 3 1 2 2 3 3 4 2 2 5	3 1 2 5

[1]In particular, we have that $(I, \not\perp, \Pi, \rho)$ with I a set of worlds, $\not\perp$ an accessibility relation, Π a collection of propositions and ρ : {formulas of an orthologic} $\to \Pi$ a function associating propositions to formulas of our logic. Under some appropriate conditions this tuple is called a Kripkean realisation of our orthologic.

Problem 6: Rational Refill

Difficulty: ★ ★ ★ ★ ★
Key words: Number Theory

The world's leading expert on everything except matter usually stores his antimatter in two spherical containers of radius A and B, respectively. However, due to some cosmic fluctuations, storing antimatter in containers of these sizes is no longer safe and the antimatter needs to be transferred to containers of different sizes. Due to its unpredictable nature, antimatter must be stored in two completely full, spherical containers of rational radii. Hence, your task is to determine if it is possible to create two spherical containers of different, rational radii, that can be completely filled with the original antimatter.

Input

- Two space-separated integers $0 \leq A, B \leq 10^6,\ B \neq 0$, the radii of the original containers.

Output

- Two line-separated non-negative rationals, the radii of the new containers. Output each rational as its numerator, followed by a '/', followed by its denominator. An integer n should be output as $n/1$. If such rationals do not exist, output `impossible`. If there are multiple answers, you may give any one of them.

Examples

Input	Output
1 2	676702467503/348671682660 415280564497/348671682660

Input	Output
1 1	impossible