

Problem : Binary Beauty - Solution

Inverting the problem, the optimal strategy is the opposite of the optimal strategy that goes from n to 1 as quickly as possible by halving or subtracting 1. Note that halving is only possible when the current length is even, so that if n is odd and not 1, the first step has to be subtracting 1. On the other hand, if n is even, we claim the optimal strategy is to halve. We show this by induction, stemming from the basis that halving is optimal when $n = 2$ as we need at least one step to reach 1.

Suppose to the contrary that for $n = 2k$ with $k > 1$ an integer, subtracting 1 is the first step in the optimal path to 1, and that for even $n < 2k$ halving is optimal. Then $2k - 1$ is odd so that we need to subtract 1 again, resulting in $2k - 2$. By the induction hypothesis we should now halve, resulting in $k - 1$ in three steps. We can however reach $k - 1$ in two steps by first halving $n = 2k$ and then subtracting 1. Thus, first subtracting 1 can never be part of an optimal solution in this case.

To solve the problem, we therefore start with n , halve if n is even and otherwise subtract 1. For the solution, we reverse this process. Note that this is the same strategy that is used in exponentiation by squaring, also known as binary exponentiation. Hence the name of the problem.